PhD proposal on
Distance constrained graph coloring

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Graph colouring is an important field of graph theory with many applications in several domains. In a graph coloring, the aim is to assign colors (labels) to the elements (vertices, edges, vertices and edges, faces, etc) of the graph with most of the time the constraint that adjacent or incident elements get disctinct colours and, depending on the type of colouring, with one or more additional constraints. In most of the cases, the goal is to minimize the number of colours used. See the book of Chartrand[3] for a comprehensive introduction to graph colouring.

The aim of this proposal is to study different types of colourings with distance constraints such as L-labeling and radio k-colouring [5, 2, 7], packing and S-packing coloring [1, 4, 8] and d-distant colouring[6] for which the constraints rely on the distance between vertices or between colors given to the elements. Our objective is to find new deep results for these graph invariants by studing them in a unified approach taking into account both types of constraints. Among the possible directions of research, one can try to find dichotomies between polynomialy solvable configurations and NP-complete ones; look at infinite graphs and find configurations that require an infinite number of colours; study the unified parameter on simple graph classes such as paths, trees, planar graphs, graph products, etc.

References

[1] B. Brešar, S. Klavžar, and D. F. Rall. On the packing chromatic number of Cartesian products, hexagonal lattice, and trees, *Discrete Applied Mathematics* **155**(17):2303–2311, 2007.

[2] R. Cada, J. Ekstein, P. Holub, O. Togni, Radio Labelings of Distance Graphs, *Discrete Applied Math.*, 161(18), pp. 2876?2884, 2013.

[3] G. Chartrand and P. Zhang. Chromatic graph theory. *CRC Press* ,2008.

[4] N. Gastineau, O. Togni, S-Packing Colorings of Cubics Graphs, Discrete Math., 339 (10), pp.2461-2470, 2016.

[5] J. R. Griggs and R. K. Yeh, Labeling graphs with a condition at distance two. *SIAM J. Discrete Math.*, 5 (1992,) 586-5995.

[6] F. Kramer, H. Kramer, A survey on the distance-colouring of graphs, Discrete Mathematics, Volume 308, Issues 2–3, 6 February 2008, Pages 422-426

[7] R. Khennoufa and O. Togni. The radio antipodal and radio numbers of the hypercube. *Ars Combinatoria*, 102 , pp. 447-461, 2011.

[8] O. Togni, On Packing Colorings of Distance Graphs, Discrete Applied Math., Volume 167, pp. 280?289, 2014.